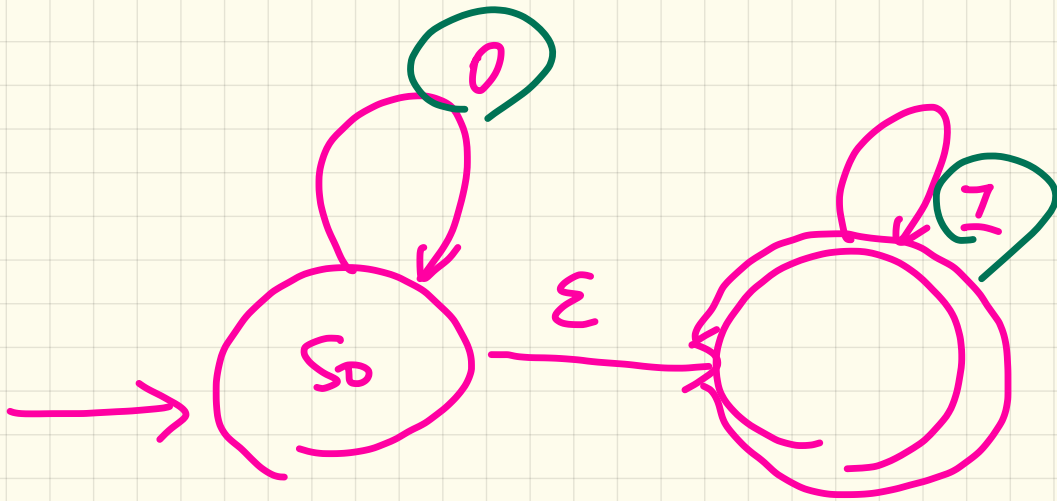
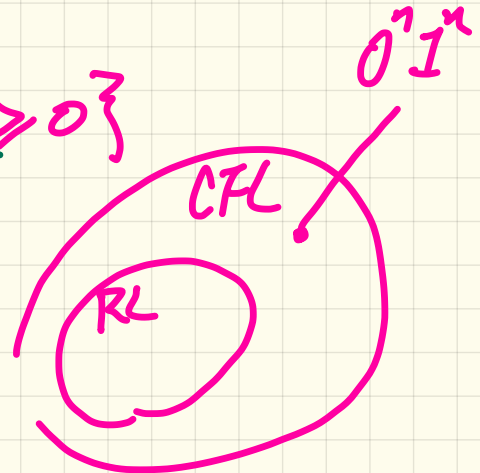


LECTURE 6

WEDNESDAY JANUARY 22



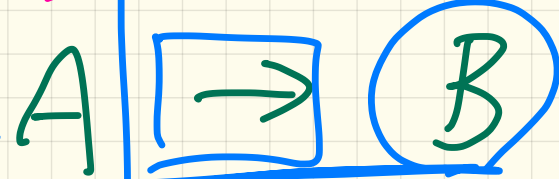
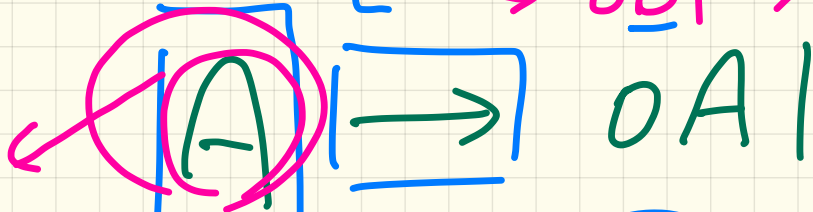
$$\{0^n 1^n \mid \underline{n} \geq 0\}$$



derivation

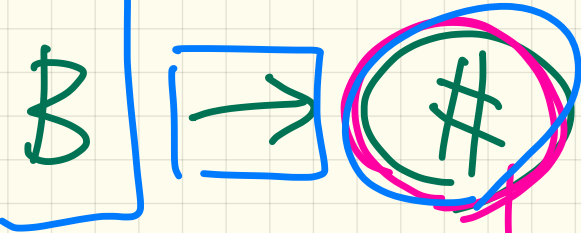
Start

$$\begin{aligned} [ & A \Rightarrow 0A1 \\ & \Rightarrow \underline{0}B1 \Rightarrow 0\underline{1}1 \end{aligned}$$

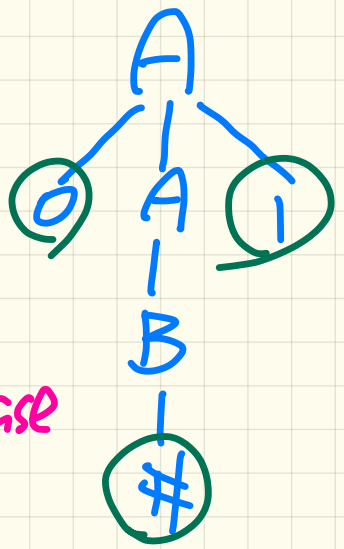


01 ∉ L

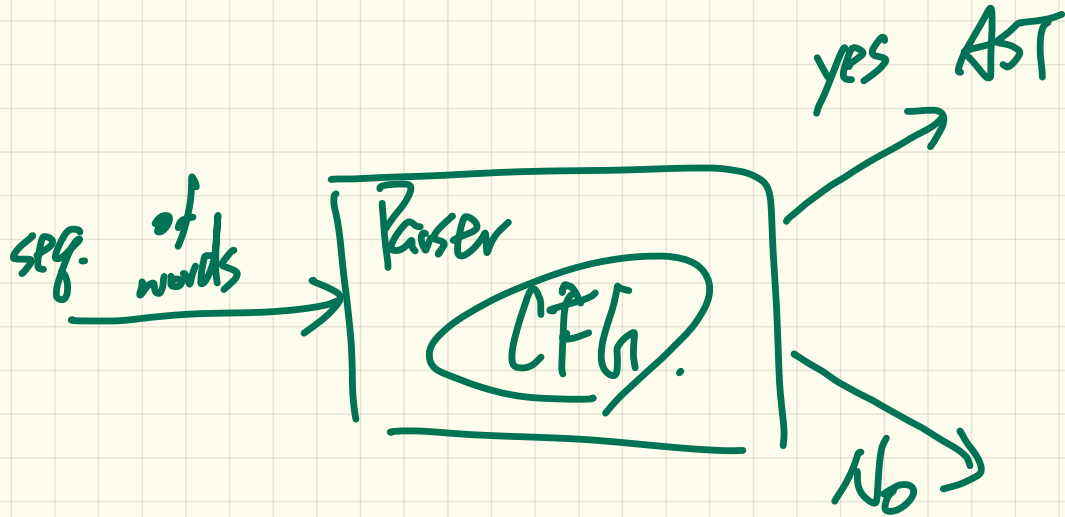
Variables  
(non-terminals)



base case



↳ RECURSIVE CASES



if (true)

$\mathcal{L}_1$ 

$$\{w \mid w \text{ is palindrome}\} =$$

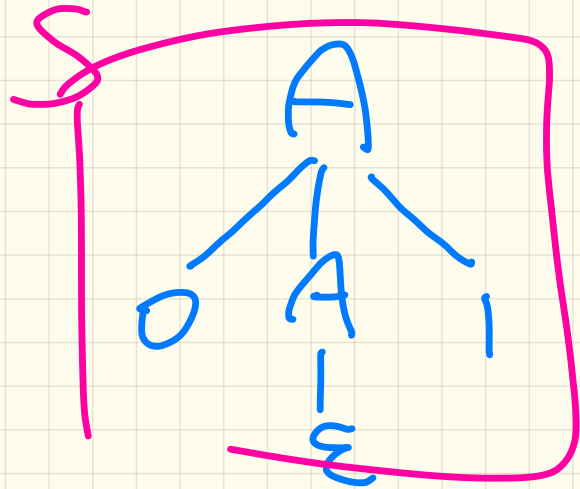
$$\{ww^R \mid w \in \Sigma^*\} \mathcal{L}_2$$

$$1 \in \mathcal{L}_1$$

$$1 \notin \mathcal{L}_2$$

A A A

A  $\rightarrow$  0 /  
unnecessary.



true + false

3 ⇒ 4

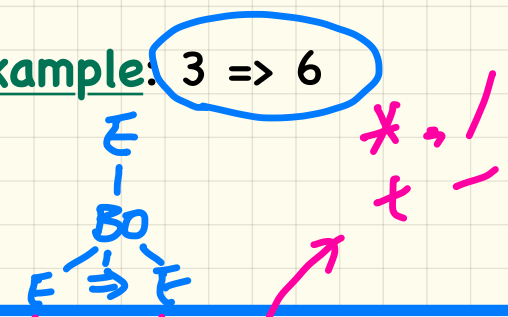
# Context-Free Grammar (CFG): Example Version 1

Expression	→	IntegerConstant
		BooleanConstant
		BinaryOp
		UnaryOp
		( Expression )
IntegerConstant	→	Digit
		Digit IntegerConstant
		- IntegerConstant
Digit	→	0   1   2   3   4   5   6   7   8   9
BooleanConstant	→	TRUE
		FALSE

Example.  $3 * 5 + 4$  } ambiguous

EXERCISE:  
draw all possible parse trees for it.

Example:  $3 \Rightarrow 6$



BinaryOp	→	Expression + Expression
		Expression - Expression
		Expression * Expression
		Expression / Expression
		Expression && Expression
		Expression    Expression
		Expression => Expression
		Expression == Expression
		Expression /= Expression
		Expression > Expression
		Expression < Expression
UnaryOp	→	! Expression



# Context-Free Grammar (CFG): Example Version 1

*Expression* → *IntegerConstant*  
| *BooleanConstant*  
| *BinaryOp*  
| *UnaryOp*  
| ( *Expression* )

*IntegerConstant* → *Digit*  
| *Digit IntegerConstant*  
| -*IntegerConstant*

*Digit* → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

*BooleanConstant* → TRUE  
| FALSE

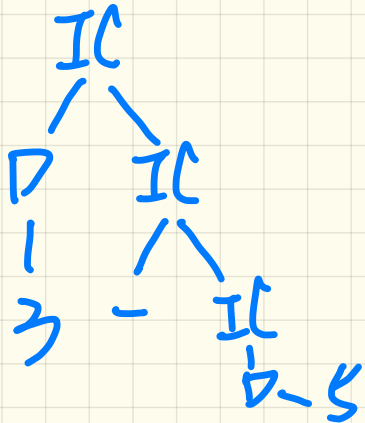
Example: 3 \* 5 + 4

*BinaryOp* → *Expression* + *Expression*  
| *Expression* - *Expression*  
| *Expression* \* *Expression*  
| *Expression* / *Expression*  
| *Expression* && *Expression*  
| *Expression* || *Expression*  
| *Expression* => *Expression*  
| *Expression* == *Expression*  
| *Expression* /= *Expression*  
| *Expression* > *Expression*  
| *Expression* < *Expression*

*UnaryOp* → ! *Expression*

# Context-Free Grammar (CFG): Example **Version 2**

<u>Expression</u>	→	ArithmeticOp   RelationalOp   <u>LogicalOp</u>   ( Expression )
IntegerConstant	→	Digit   <u>Digit IntegerConstant</u>   -IntegerConstant
Digit	→	0   1   2   3   4   5   6   7   8   9
BooleanConstant	→	TRUE   FALSE



Example: (1 + 2) => (5 / 4)

ArithmeticOp	→	ArithmeticOp + ArithmeticOp   ArithmeticOp - ArithmeticOp   ArithmeticOp * ArithmeticOp   ArithmeticOp / ArithmeticOp   ( ArithmeticOp )   IntegerConstant
RelationalOp	→	ArithmeticOp == ArithmeticOp   ArithmeticOp /= ArithmeticOp   ArithmeticOp > ArithmeticOp   ArithmeticOp < ArithmeticOp
LogicalOp	→	LogicalOp && LogicalOp   LogicalOp    LogicalOp   <u>LogicalOp =&gt; LogicalOp</u>   ! LogicalOp   ( LogicalOp )   RelationalOp   BooleanConstant

# Context-Free Grammar (CFG): Example Version 2

Expression	→	ArithmeticOp   RelationalOp   LogicalOp   ( Expression )
IntegerConstant	→	Digit   Digit IntegerConstant   -IntegerConstant
Digit	→	0   1   2   3   4   5   6   7   8   9
BooleanConstant	→	TRUE   FALSE

Example: ( 1 + 2 ) / ( 5 - ( 2 + 3 ) )  
↳ a valid step.

ArithmeticOp	→	ArithmeticOp + ArithmeticOp   ArithmeticOp - ArithmeticOp   ArithmeticOp * ArithmeticOp   ArithmeticOp / ArithmeticOp   ( ArithmeticOp )   IntegerConstant
RelationalOp	→	ArithmeticOp == ArithmeticOp   ArithmeticOp /= ArithmeticOp   ArithmeticOp > ArithmeticOp   ArithmeticOp < ArithmeticOp
LogicalOp	→	LogicalOp && LogicalOp   LogicalOp    LogicalOp   LogicalOp => LogicalOp   ! LogicalOp   ( LogicalOp )   RelationalOp   BooleanConstant

Choice:  
report this as compilation error.

# Context-Free Grammar (CFG): Example Version 2

<i>Expression</i>	→	<i>ArithmeticOp</i>   <i>RelationalOp</i>   <i>LogicalOp</i>   ( <i>Expression</i> )
<i>IntegerConstant</i>	→	<i>Digit</i>   <i>Digit IntegerConstant</i>   - <i>IntegerConstant</i>
<i>Digit</i>	→	0   1   2   3   4   5   6   7   8   9
<i>BooleanConstant</i>	→	TRUE   FALSE

Example:  $3 * 5 + 4$

↳ exercise: parse trees

<i>ArithmeticOp</i>	→	<i>ArithmeticOp</i> + <i>ArithmeticOp</i>   <i>ArithmeticOp</i> - <i>ArithmeticOp</i>   <i>ArithmeticOp</i> * <i>ArithmeticOp</i>   <i>ArithmeticOp</i> / <i>ArithmeticOp</i>   ( <i>ArithmeticOp</i> )   <i>IntegerConstant</i>
<i>RelationalOp</i>	→	<i>ArithmeticOp</i> == <i>ArithmeticOp</i>   <i>ArithmeticOp</i> /= <i>ArithmeticOp</i>   <i>ArithmeticOp</i> > <i>ArithmeticOp</i>   <i>ArithmeticOp</i> < <i>ArithmeticOp</i>
<i>LogicalOp</i>	→	<i>LogicalOp</i> && <i>LogicalOp</i>   <i>LogicalOp</i>    <i>LogicalOp</i>   <i>LogicalOp</i> => <i>LogicalOp</i>   ! <i>LogicalOp</i>   ( <i>LogicalOp</i> )   <i>RelationalOp</i>   <i>BooleanConstant</i>

$$u \underbrace{A}_v \xRightarrow{A \rightarrow w} u \underbrace{w}_v$$

$$S \rightarrow (S) \mid \underline{SS} \mid \varepsilon$$
$$S \underline{(}$$